 LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

**M.Sc.** DEGREE EXAMINATION - **MATHEMATICS**

THIRD SEMESTER – **APRIL 2012**

# MT 3810 - TOPOLOGY

Date : 21-04-2012 Dept. No. Max. : 100 Marks

Time : 1:00 - 4:00

**Answer all questions. All questions carry equal marks. 5 x 20 = 100 marks**

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| **01)** | **(a)** | (i) | Let X be a non-empty set and let d be a real function of ordered pairs of elements of X which satisfies the following conditions.    2. Show that d is a metric on X.   **(or)** |
|  |  | (ii) | Let X be a metric space. Prove that a subset G of X is open it is a union of open spheres. (5) |
|  | **(b)** | (i) | Let X be a metric space, and let Y be a subspace of X. Prove that Y is complete iff Y is closed. |
|  |  | (ii) | State and prove Cantor’s Intersection Theorem. |
|  |  | (iii) | State and prove Baire’s Theorem. (6+5+4)  **(or)** |
|  |  | (iv) | Let X and Y be metric spaces and let f be a mapping of X into Y. Prove that f is continuous at  and f is continuous is open in X whenever G is open in Y. (15) |
| **02)** | **(a)** | (i) | Prove that every separable metric space is second countable.  **(or)** |
|  |  | (ii) | Define a topology on a non-empty set  with an example. Let  be a topological space and  be an arbitrary subset of . Show that each neighbourhood of intersects . (5) |
|  | **(b)** | (i) | Show that any continuous image of a compact space is compact. |
|  |  | (ii) | Prove that any closed subspace of a compact space is compact. |
|  |  | (iii) | Give an example to show that a compact subspace of a compact space need not be closed. (6+6+3)  **(or)** |
|  |  | (iv) | Show that a topological space is compact, if every subbasic open cover has a finite subcover. (15) |
| **03)** | **(a)** | (i) | State and prove Tychnoff’s Theorem.  **(or)** |
|  |  | (ii) | Show that a metric space is compact if it is complete and totally bounded. (5) |
|  | **(b)** | (i) | Prove that in a sequentially compact space, every open cover has a Lebesgue’s number. |
|  |  | (ii) | Prove that every sequentially compact metric space is totally bounded. (10+5)  **(or)** |
|  |  | (iii) | State and prove Ascoli’s Theorem. (15) |
| **04)** | **(a)** | (i) | Show that every subspace of Hausdorff space is also Hausdorff.  **(or)** |
|  |  | (ii) | Prove that every compact Hausdorff Space is normal. (5) |
|  | **(b)** | (i) | Prove that the product of any non-empty class of Hausdorff Spaces is a Hausdorff Space. |
|  |  | (ii) | Prove that every compact subspace of a Hausdorff space is closed. |
|  |  | (iii) | Show that a one-to-one continuous mapping of a compact space onto a Hausdorff Space is a homeomorphism. (6+4+5)  **(or)** |
|  |  | (iv) | If X is a second countable normal space, prove that there exists a homeomorphism f of X onto a subspace of and X is therefore metrizable. (15) |
| **05)** | **(a)** | (i) | Prove that any continuous image of a connected space is connected.  **(or)** |
|  |  | (ii) | Show that the components of a totally disconnected space are its points. (5) |
|  | **(b)** | (i) | Show that the product of any non-empty class of connected spaces is connected. |
|  |  | (ii) | Let X be a compact Hausdorff Space. Show that X is totally disconnected, iff it has open base whose sets are also closed. (6+9)  **(or)** |
|  |  | (iii) | State and prove the Weierstrass Approximation Theorem. (15) |